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# V 型三能级系统中 Raman 过程诱导的无反转激光

胡响明
（华中师范大学应用物理研究所，武汉 430070）

张千年
（华中师范大学信息技术系，武汉 430070）

**提要**　研究了封闭的 V 型三能级激光系统。该系统利用外加双色泵浦场通过 Raman 过程耦合两个高能级来建立原子相干，激光跃迁发生在一个高能级与低能级之间。结果表明该系统可以建立裸态无反转激光，其增益来源于缀饰态原子相干。

**关键词**　Raman 过程，原子相干，无反转激光

## 1　引　言

利用外场诱导的原子相干[1~10]，或者利用不同通道的量子干涉[11]，可以建立无反转激光。实验上已经探测到无反转激光的瞬间放大信号[12]。原子相干和量子干涉诱导的无反转激光，具有线宽窄[13]、振幅压缩[14]等统计特性。因此，目前无反转激光是激光物理和量子光学领域的研究热点之一。

无反转激光的增益机制依赖于不同的激光模型[9]，反转的含义依赖于所用基矢的选择[3~8,10]。常用的有裸态基矢和缀饰态基矢. 裸态基矢是孤立原子系统的本征矢，而缀饰态基矢是原子和耦合原子的光场一起构成的整个系统的本征矢。一些系统可以呈现裸态无反转激光[8~10]，其增益或者来源于缀饰态布居反转[9,10]，或者来源于缀饰态原子相干[8]；一些系统可以呈现任意态（裸态和缀饰态）无反转激光[2~7]，其增益来源于缀饰态原子相干[6]。上述原子相干均由单光子过程产生。

本文考虑封闭的 V 型三能级激光系统. 相干泵浦是利用双色泵浦场的 Raman 过程来建立原子相干，泵浦场与原子发生相互作用的单光子失谐很大，以致单光子过程可以忽略，激光跃迁发生在一个高能级与低能级之间。结果表明，系统可以呈现裸态无反转激光，无反转激光的增益来源于缀饰态原子相干。

## 2　模型与方程

考虑封闭的 V 型三能级系统，如图 1($a$) 所示。原子的两个高能级与低能级分别为 $|1\rangle$，$|2\rangle$ 和 $|3\rangle$，能级 $|1\rangle$ 和 $|2\rangle$ 之间通过一个双色泵浦场建立原子相干。双色泵浦场耦合到原子的

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|1〉－|3〉和|2〉－|3〉跃迁的单光子失谐很大，但能级|1〉和|2〉之间发生双光子共振。此时单光子过程的作用可以忽略，双光子过程起主要作用，因此泵浦场与原子的相互作用可用等效哈密顿来描述[15,16]。这个等效哈密顿描述的泵浦过程是一个 Raman 过程。激光跃迁共振地发生在能级|2〉和|3〉之间。整个系统相互作用的哈密顿为：

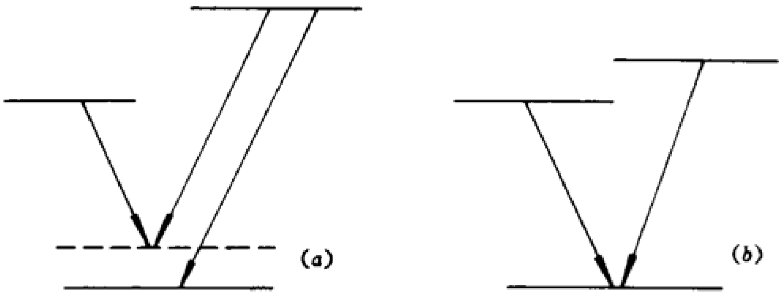

Fig. 1　Energy level diagram for inversionless lasing induced by Raman Process

(a) V-type three-level system driven by a Raman process; (b) three-level system in the dressed state basis

$$V = - G_1G_2^* |1\rangle\langle 2| - G_1^*G_2|2\rangle\langle 1| - g|2\rangle\langle 3| - g^* |3\rangle\langle 2| \tag{1}$$

其中，$2G_1$ 和 $2G_2$ 分别是双色泵浦场耦合到原子跃迁|1〉－|3〉和|2〉－|3〉的拉比频率，$2G_1G_2^*$ 是由等效哈密顿描述的双色泵浦场与原子之间发生的 Raman 耦合的拉比频率，$2g$ 是探测场与原子耦合的拉比频率。

系统的约化密度矩阵方程为：

$$\begin{aligned}
\rho_{32} &= - \gamma_{23}\rho_{32} + ig^* (\rho_{22} - \rho_{33}) - iG_1G_2^* \rho_{31} \\
\rho_{31} &= - \gamma_{13}\rho_{31} + ig^* \rho_{21} - iG_1^* G_2\rho_{32} \\
\rho_{21} &= - \gamma_{12}\rho_{21} + iG_1^* G_2(\rho_{11} - \rho_{22}) + ig\rho_{31} \\
\rho_{33} &= - 2(\lambda_1 + \lambda_2)\rho_{33} + 2\gamma_1\rho_{11} + 2\gamma_2\rho_{22} + i(g^* \rho_{23} - g\rho_{32}) \\
\rho_{11} &= - 2\gamma_1\rho_{11} + 2\lambda_1\rho_{33} + i(G_1G_2^* \rho_{21} - G_1^* G_2\rho_{12})
\end{aligned} \tag{2}$$

其中，　$\gamma_{13} = \gamma_1 + \lambda_1 + \lambda_2$，　$\gamma_{23} = \gamma_2 + \lambda_1 + \lambda_2$，　$\gamma_{12} = \gamma_1 + \gamma_2$

$2\gamma_1$ 和 $2\gamma_2$ 是原子分别从能级|1〉和|2〉到|3〉的自发辐射衰减速率，$2\lambda_1$ 和 $2\lambda_2$ 是原子从能级|3〉分别到能级|1〉和|2〉的非相干泵浦速率。

利用归一化条件 $\rho_{11} + \rho_{22} + \rho_{33} = 1$，求解方程组(2)，得到原子相干 $\rho_{32}$ 和 $\rho_{21}$ 以及布居几率 $\rho_{ii}(i = 1,2,3)$ 分别为：

$$\begin{aligned}
&\rho_{32} = ig^* [P_{23}(\rho_{22} - \rho_{33}) + Q_{23}(\rho_{11} - \rho_{22})] \\
&\rho_{21} = iG_1G_2^* [P_{12}(\rho_{11} - \rho_{22}) + Q_{23}(\rho_{22} - \rho_{33})] \\
&\rho_{ii} = \frac{D_{ii}}{D}, \qquad i = 1,2,3 \\
&D_{11} = X_{12}X_{23} - X_{22}X_{13}, \qquad D_{22} = X_{21}X_{13} - X_{11}X_{23} \\
&D_{33} = X_{11}X_{22} - X_{12}X_{21}, \qquad D = D_{11} + D_{22} + D_{33} \\
&X_{11} = \gamma_1 + |g|^2Q_{23}, \qquad X_{12} = \gamma_2 + |g|^2(P_{23} - Q_{23}) \\
&X_{13} = - (\lambda_1 + \lambda_2) - |g|^2P_{23}, \quad X_{21} = - \gamma_1 - |G_1G_2|^2P_{12} \\
&X_{22} = |G_1G_2|^2(P_{12} - Q_{12}), \qquad X_{23} = \lambda_1 + |G_1G_2|^2Q_{12} \\
&P_{23} = U(\gamma_{12}\gamma_{13} + |g|^2), \qquad Q_{23} = U|G_1G_2|^2 \\
&P_{12} = U(\gamma_{13}\gamma_{23} + |G_1G_2|^2), \quad Q_{12} = U|g|^2
\end{aligned} \tag{3}$$

$$U = (\gamma_{12}\gamma_{23}\gamma_{13} + \gamma_{12}|G_1G_2|^2 + \gamma_{23}|g|^2)^{-1}$$

略去一些无关的比例常数，激光的增益为[8]：

$$G = \mathrm{Im}(\rho_{32}/g^*) = P_{23}(\rho_{22} - \rho_{33}) + Q_{23}(\rho_{11} - \rho_{22}) \tag{4}$$

其中，第一和第二项

$$G_B^P = P_{23}(\rho_{22} - \rho_{33}), \qquad G_B^C = Q_{23}(\rho_{11} - \rho_{22}) \tag{5}$$

分别是裸态原子布居几率差 $\rho_{22} - \rho_{33}$ 和裸态原子相干 $\rho_{21}$ 的贡献。

## 3 无反转激光

考虑弱探测场，按通常的方式保留原子相干到$g$的一次幂，保留布居几率到$g$的零次幂，得到

$$\begin{aligned}
\rho_{32}^1 &= \frac{ig^*}{|G_1G_2|^2 + \gamma_{13}\gamma_{23}}\left[\gamma_{13}(\rho_{22}^0 - \rho_{33}^0) + \frac{|G_1G_2|^2}{\gamma_{12}}(\rho_{11}^0 - \rho_{22}^0)\right] \\
\rho_{11}^0 &= \frac{1}{D_0}\left[\lambda_1\gamma_2 + (\lambda_1 + \lambda_2)\frac{|G_1G_2|^2}{\gamma_{12}}\right] \\
\rho_{22}^0 &= \frac{1}{D^0}\left[\lambda_2\gamma_1 + (\lambda_1 + \lambda_2)\frac{|G_1G_2|^2}{\gamma_{12}}\right] \\
\rho_{33}^0 &= \frac{1}{D^0}\left[\gamma_1\gamma_2 + (\gamma_1 + \gamma_2)\frac{|G_1G_2|^2}{\gamma_{12}}\right]
\end{aligned} \tag{6}$$

其中，
$$D^0 = \gamma_1\gamma_2 + \lambda_1\gamma_2 + \lambda_2\gamma_1 + \left[(\lambda_1 + \lambda_2) + 2(\gamma_1 + \gamma_2)\frac{|G_1G_2|^2}{\gamma_{12}}\right]$$

对于强场泵浦，$|G_1G_2| \gg \gamma_i, \gamma_{ij}, \lambda_i$，裸态无反转 $\rho_{22}^0 < \rho_{33}^0$ 的条件是

$$\lambda_1 + \lambda_2 < \gamma_1 + \gamma_2 \tag{7}$$

光放大条件（即增益为正，$G > 0$）的要求是

$$\lambda_1 + \lambda_2 > \gamma_1 + \gamma_2 - \frac{\lambda_1\gamma_2 - \lambda_2\gamma_1}{\gamma_1 + \lambda_1 + \lambda_2}, \qquad \lambda_1\gamma_2 > \lambda_2\gamma_1 \tag{8}$$

因此，无反转光放大的条件则为

$$\gamma_1 + \gamma_2 - \frac{\lambda_1\gamma_2 - \lambda_2\gamma_1}{\gamma_1 + \lambda_1 + \lambda_2} < \lambda_1 + \lambda_2 < \gamma_1 + \gamma_2, \qquad \lambda_1\gamma_2 > \lambda_2\gamma_1 \tag{9}$$

## 4 缀饰态分析

为了清楚地表明无反转激光增益的物理起源，这里对缀饰态进行分析。在原子与双色泵浦场的 Raman 相互作用哈密顿

$$H_d = - G_1G_2^*|1\rangle\langle 2| - G_1^*G_2|2\rangle\langle 1| \tag{10}$$

的对角化表像中，态矢〔如图 1($b$) 所示〕可写为裸态态矢的线性组合（为了简单，假定$\Omega = G_1G_2^*$为实数）

$$|+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle), \qquad |-\rangle = \frac{1}{\sqrt{2}}(-|1\rangle + |2\rangle), \qquad |0\rangle = |3\rangle \tag{11}$$

以下反复使用的裸态与缀饰态中密度矩阵的关系为

$$
\begin{aligned}
&\rho_{11} = \frac{1}{2}(\rho_{++} + \rho_{--} - \rho_{+-} - \rho_{-+}), && \rho_{22} = \frac{1}{2}(\rho_{++} + \rho_{--} + \rho_{+-} + \rho_{-+}) \\
&\rho_{12} = \frac{1}{2}(\rho_{++} - \rho_{--} + \rho_{+-} - \rho_{-+}), && \rho_{31} = \frac{1}{\sqrt{2}}(\rho_{0+} - \rho_{0-}) \\
&\rho_{32} = \frac{1}{\sqrt{2}}(\rho_{0+} + \rho_{0-}) && \rho_{00} = \rho_{33} \\
&\rho_{++} + \rho_{--} = \rho_{11} + \rho_{22} && \rho_{++} - \rho_{--} = \rho_{12} + \rho_{21} \\
&\rho_{+-} + \rho_{-+} = \rho_{22} - \rho_{11} && \rho_{+-} - \rho_{-+} = \rho_{12} - \rho_{21}
\end{aligned}
\tag{12}
$$

将这些关系式代入方程组(2) 中的前两个方程，得到关于 $\rho_{0+}$ 和 $\rho_{0-}$ 的方程组：

$$
\begin{aligned}
\rho_{0+} &= -\left[\frac{1}{2}(\gamma_{13} + \gamma_{23}) + i\Omega\right]\rho_{0+} - \frac{1}{2}(\gamma_{23} - \gamma_{13})\rho_{0-} + \frac{ig^*}{\sqrt{2}}(\rho_{++} + \rho_{-+} - \rho_{00}) \\
\rho_{0-} &= -\frac{1}{2}(\gamma_{23} - \gamma_{13})\rho_{0+} - \left[\frac{1}{2}(\gamma_{13} + \gamma_{23}) - i\Omega\right]\rho_{0-} + \frac{ig^*}{\sqrt{2}}(\rho_{--} + \rho_{+-} - \rho_{00})
\end{aligned}
\tag{13}
$$

解出 $\rho_{0+}$ 和 $\rho_{0-}$，于是得到信号场之源 $\rho_{32}$

$$
\rho_{32} = \frac{ig^*}{\Omega^2 + \gamma_{13}\gamma_{23}}\left[(\gamma_{13} + \Omega^2 Q_{12})\left(\frac{\rho_{++} + \rho_{--}}{2} - \rho_{00}\right) - \left(\Omega^2 P_{12} - \frac{\gamma_{13}}{2} - \frac{1}{2}\Omega^2 Q_{12}\right)(\rho_{+-} + \rho_{-+})\right]
\tag{14}
$$

同样略去无关的比例常数，缀饰态布居几率差和原子相干 $\rho_{0+}$ 与 $\rho_{0-}$ 对增益的贡献分别是[17]

$$
\begin{aligned}
G_D^P &= \frac{\gamma_{13} + \Omega^2 Q_{12}}{\Omega^2 + \gamma_{13}\gamma_{23}}\left(\frac{\rho_{++} + \rho_{--}}{2} - \rho_{00}\right) \\
G_D^C &= -\frac{2\Omega^2 P_{12} - \gamma_{13} - \Omega^2 Q_{12}}{2(\Omega^2 + \gamma_{13}\gamma_{23})}(\rho_{+-} + \rho_{-+})
\end{aligned}
\tag{15}
$$

至此，在缀饰态中的计算未作任何近似. 按通常的方式考虑弱探测场，将 $G_D^P$ 和 $G_D^C$ 保留到 $g$ 的零次幂[2~7]，并利用(12) 式简化为：

$$
\begin{aligned}
G_D^P &= \frac{\gamma_{13}}{\Omega^2 + \gamma_{13}\gamma_{23}}\left(\frac{\rho_{11}^0 + \rho_{22}^0}{2} - \rho_{33}^0\right) \\
G_D^C &= \frac{2\Omega^2 - \gamma_{12}\gamma_{13}}{2\gamma_{12}(\Omega^2 + \gamma_{13}\gamma_{23})}(\rho_{11}^0 - \rho_{22}^0)
\end{aligned}
\tag{16}
$$

对于强场泵浦情形，有：

$$
\rho_{11}^0 + \rho_{22}^0 \approx 2\rho_{22}^0 \qquad \rho_{11}^0 - \rho_{22}^0 = \frac{1}{D^0}(\lambda_1\gamma_2 - \lambda_2\gamma_1)
\tag{17}
$$

在无反转 $\rho_{22}^0 < \rho_{33}^0$ 条件下，$G_D^P < 0$，然而当满足

$$
\lambda_1\gamma_2 > \lambda_2\gamma_1
\tag{18}
$$

时，原子相干的贡献为正，$G_D^C > 0$，因此，只要原子相干和布居几率差二者的总贡献为正，即

$$
G = G_D^P + G_D^C > 0, \quad 或\ G_D^C > -G_D^P
\tag{19}
$$

系统呈现无反转光放大。可见，无反转激光的增益来源于缀饰态原子相干。

总之，本文建议了在封闭的 V 型三能级系统中采用双色泵浦场的 Raman 过程产生缀饰态原子相干来建立无反转激光的方案。Ba 原子可作为这个方案的一个例子，其中 $6s^2\,{}^1S_0$ 基态作为态 $|3\rangle$，$6s6p\,{}^3P_1$ 和 $6s6p\,{}^1P_1$ 两个激发态分别作为态 $|1\rangle$ 和态 $|2\rangle$。

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# Lasing without Inversion Induced by Raman Process in a V-type Three-level System

Hu Xiangming

*(Institute of Applied Physics, Huazhong Normal University, Wuhan* 430070)

Zhang Gannian

*(Department of Information Technology, Huazhong Normal University, Wuhan* 430070)

**Abstract** In this paper, a closed V-type three-level system is studied. In the system, atomic coherence is induced by a Raman process of coupling the two upper levels, and lasing occurs at the transition between either of the upper levels and the lower level. It is shown that this system can exhibit lasing without inversion in the bare state, and the gain arises from the atomic coherence in the dressed state.

**Key words** Raman process, atomic coherence, lasing without inversion