中国激光
第14卷 第9期

带电粒子的自有场对自由电子激光器增益的影响

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提要: 研究了考虑带电粒子自有场时自由电子激光器的增益，并与忽略带电粒子自有场情况下所得的结果相比较。结果表明，自有场对激光器增益有一定的作用。

Effect of charged particle self-field on the gain of FEL's

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Abstract: Considering the condition of charged particle self-field the gain of FEL is studied and compared to the results obtained by neglecting the self-field. It is shown that self-field has some effects on the gain of the FEL's.

一、引 言

在自由电子激光器的理论研究中，有一类称为单粒子轨道理论的经典理论处理法[1~6]。这类方法虽然一般不考虑集体效应，但由于描述系统运动状态的概念比较清晰，且较容易获得解析结果。但在目前的单粒子轨道理论分析中，一般只考虑外加静周期磁场和辐射场与相对论电子的相互作用，而没有考虑带电粒子运动本身产生的自有场(或称自生场)的作用。由于运动的带电粒子必然附有电磁场[7]，所以已有的解析或计算结果与实际的自由电子激光器还存在一定的差距。本文在原有的单粒子轨道理论基础上，考虑了带电粒子自有场，利用逐阶近似分析法获得了在这种条件下的激光增益表达式，并与忽略自有场情况下的理论分析结果相比较。

二、方程建立

带电粒子运动时，它所产生的电磁场可根据下列公式解得

$$\varphi(x, y, z, t) = \iiint \frac{\rho\left(x', y', z', t-\frac{r}{c}\right)d\tau'}{r} \quad (1)$$

$$A(x, y, z, t) = \frac{1}{c}\iiint \frac{j\left(x', y', z', t-\frac{r}{c}\right)d\tau'}{r} \quad (2)$$

收稿日期: 1986年4月26日。

式中φ和$\boldsymbol{A}$分别为带电粒子在(x, y, z)点位置t时刻的总电磁场标量势和矢量势；ρ是带电粒子的电荷密度；(x', y', z')为t时刻前带电粒子位置坐标；$\boldsymbol{r}$是(x', y', z')到(x, y, z)之间径向距离；c是光在真空中速度；$\boldsymbol{j}$是粒子运动的电流密度。(1)和(2)式可由推迟势公式求出[7]，

$$\varphi=\frac{q}{\sqrt{r_{\perp}^{2}\left(1-\frac{v_{0}^{2}}{c^{2}}\right)+r_{/\!/}^{2}}} \tag{3}$$

$$\boldsymbol{A}=\frac{q\boldsymbol{v}_{0}}{c\sqrt{r_{\perp}^{2}\left(1-\frac{v_{0}^{2}}{c^{2}}\right)+r_{/\!/}^{2}}} \tag{4}$$

式中$\boldsymbol{r}_{/\!/}$和$r_{\perp}$分别为$\boldsymbol{r}$在平行$\boldsymbol{v}_0$和垂直$\boldsymbol{v}_0$方向的分量；v_0是粒子的运动速度，q为粒子的电荷。由(3)、(4)式得带电粒子运动的电磁场（自有场）为[注]

$$\boldsymbol{E}_{f}=\frac{q(1-\beta_{0}^{2})\boldsymbol{r}}{\left[r_{\perp}^{2}(1-\beta_{0}^{2})+r_{/\!/}^{2}\right]^{\frac{3}{2}}} \tag{5}$$

$$\boldsymbol{B}_{f}=\frac{q(1-\beta_{0}^{2})\boldsymbol{v}_{0}\times\boldsymbol{r}}{c\left[r_{\perp}^{2}(1-\beta_{0}^{2})+r_{/\!/}^{2}\right]^{\frac{3}{2}}} \tag{6}$$

式中$\beta_0=\frac{v_0}{c}$，由于$v_0\to c$时整个电磁场集中在垂直于v_0的平面内[7]，则有$\boldsymbol{r}_{\perp}\approx\boldsymbol{r}$，$\boldsymbol{r}_{/\!/}\approx0$，设带电粒子为相对论电子，由(5)、(6)式得相对论电子运动中产生的自有场为

$$\boldsymbol{E}_{fe}=\frac{|e|(1-\beta_{0}^{2})\boldsymbol{r}}{r^{3}(1-\beta_{0}^{2})^{\frac{3}{2}}} \tag{7}$$

$$\boldsymbol{B}_{fe}=\frac{|e|(1-\beta_{0}^{2})\boldsymbol{v}_{0}\times\boldsymbol{r}}{cr^{3}(1-\beta_{0}^{2})^{\frac{3}{2}}} \tag{8}$$

假定相对论电子通过的Wiggler的周期磁场为

$$\boldsymbol{B}_{m}(z)=B_{0}(\cos(k_{0}z\boldsymbol{x}+\sin k_{0}z\boldsymbol{y}) \tag{9}$$

式中B_0是周期磁场的振幅；$k_0=2\pi/\lambda_0$，λ_0是周期长度。相对论电子在摆动器内产生的光辐射场为

$$\boldsymbol{E}_{r}=E_{r0}[\cos(k_{r}z-\omega_{r}t+\Psi_{r})\boldsymbol{x}-\sin(k_{r}z-\omega_{r}t+\Psi_{r})\boldsymbol{y}] \tag{10}$$

$$\boldsymbol{B}_{r}=\frac{E_{r0}}{c}[\sin(k_{r}z-\omega_{r}t+\Psi_{r})\boldsymbol{x}+\cos(k_{r}z-\omega_{r}t+\Psi_{r})\boldsymbol{y}] \tag{11}$$

式中E_{r0}是辐射场的电场振幅；k_r是辐射场波矢，$k_r=2\pi/\lambda_r$，λ_r为辐射波长；ω_r为辐射场圆频率，$\omega_r=k_rc$；Ψ_r为辐射场初相位。由于相对论电子在摆动器磁场作用下作螺旋形回旋运动[5]，则由(7)、(8)式表示的相对论电子自有场可进一步表示为

$$\boldsymbol{E}_{fe}=E_{f0}[\sin(k_{f}z-\omega_{f}t+\Psi_{f})\boldsymbol{x}+\cos(k_{f}z-\omega_{f}t+\Psi_{f})\boldsymbol{y}] \tag{12}$$

$$\boldsymbol{B}_{fe}=\frac{E_{f0}}{c}[-\cos(k_{f}z-\omega_{f}t+\Psi_{f})\boldsymbol{x}+\sin(k_{f}z-\omega_{f}t+\Psi_{f})\boldsymbol{y}] \tag{13}$$

式中E_{f0}为自有场的电场振幅，$E_{f0}=\frac{|e|(1-\beta_0^2)}{r_0^2(1-\beta_0^2)^{\frac{3}{2}}}$，$\boldsymbol{r}_0$是径矢$\boldsymbol{r}$的模量；$k_f$为自有场波矢，$k_f=2\pi/\lambda_f$，$\lambda_f$为自有场回旋周期长度；$\omega_f$为自有场圆频率，$\omega_f=k_fc$；$\Psi_f$为自有场初相位。

相对论电子在摆动器内运动规律由洛仑兹力方程和能量方程描述[6,8]

$$\frac{d}{dt}(\gamma\boldsymbol{\beta})=-\frac{|e|}{m_{0}c}(\boldsymbol{E}+c\boldsymbol{\beta}\times\boldsymbol{B}) \tag{14}$$

$$\frac{d}{dt}\gamma=-\frac{|e|}{m_{0}c}\boldsymbol{\beta}\cdot\boldsymbol{E} \tag{15}$$

式中γ为相对论因子，$\gamma=(1-\beta^2)^{-\frac{1}{2}}$，$\boldsymbol{\beta}=\frac{\boldsymbol{v}}{c}$，$\boldsymbol{v}$是相对论电子运动速度；$m_0$为电子静止质量；$\boldsymbol{E}=\boldsymbol{E}_r+\boldsymbol{E}_{fe}$，$\boldsymbol{B}=\boldsymbol{B}_r+\boldsymbol{B}_{fe}+\boldsymbol{B}_{m}$。

三、基本方程分析

(14)式的三个直角坐标分量方程为

［注］ 运动的带电粒子产生的电磁场原则上分两部分，一部分依附着带电粒子称为带电粒子的自有场；另一部分随运动速度的变化形成辐射场，由(5)、(6)式列出的电磁场不包括辐射场，而是自有场。有关运动的带电粒子电磁场的详细讨论参看文献[7]第五章内容。

$$\frac{d}{dt}(\gamma\beta_x)=-\frac{|e|}{m_0c}\Big[E_{r0}\cos\xi_r+E_{f0}\sin\xi_f-c\beta_z\Big(B_0\sin k_0z+\frac{E_{r0}}{c}\cos\xi_r+\frac{E_{f0}}{c}\sin\xi_f\Big)\Big] \tag{16}$$

$$\frac{d}{dt}(\gamma\beta_y)=-\frac{|e|}{m_0c}\Big[-E_{r0}\sin\xi_r+E_{f0}\cos\xi_f+c\beta_z\Big(B_0\cos k_0z+\frac{E_{r0}}{c}\sin\xi_r-\frac{E_{f0}}{c}\cos\xi_f\Big)\Big] \tag{17}$$

$$\frac{d}{dt}(\gamma\beta_z)=-\frac{|e|}{m_0c}\Big[c\beta_x\Big(B_0\sin k_0z+\frac{E_{r0}}{c}\cos\xi_r+\frac{E_{f0}}{c}\sin\xi_f\Big)-c\beta_y\Big(B_0\cos k_0z+\frac{E_{r0}}{c}\sin\xi_r-\frac{E_{f0}}{c}\cos\xi_f\Big)\Big] \tag{18}$$

式中 $\xi_r=k_rz-\omega_rt+\Psi_r$, $\xi_f=k_fz-\omega_ft+\Psi_r$, 分别对上面三式 t 积分,并用下面关系

$$\left.\begin{aligned}v_x&=v\sin\alpha\sin\theta=v_\perp\sin\theta\\ v_y&=-v\sin\alpha\cos\theta=-v_\perp\cos\theta\\ v_z&=v\cos\alpha\end{aligned}\right\} \tag{19}$$

得

$$\gamma\beta_x=\frac{2|e|E_{r0}}{m_0\omega_rc}\sin\xi_r-\frac{2|e|E_{f0}}{m_0\omega_fc}\cos\xi_f-\frac{|e|B_0}{m_0\omega_0}\cos k_0z+C_x \tag{20}$$

$$\gamma\beta_y=\frac{2|e|E_{r0}}{m_0\omega_rc}\cos\xi_r+\frac{2|e|E_{f0}}{m_0\omega_fc}\sin\xi_f-\frac{|e|B_0}{m_0\omega_0}\sin k_0z+C_y \tag{21}$$

$$\gamma\beta_z=\frac{|e|E_{r0}\,\mathrm{tg}\,\alpha}{m_0\omega_rc}\cos(\xi_r+\theta)+\frac{|e|E_{f0}\,\mathrm{tg}\,\alpha}{m_0\omega_fc}\sin(\xi_f-\theta)-\frac{|e|B_0\,\mathrm{tg}\,\alpha}{m_0\omega_0}\sin(k_0z-\theta)+C_z \tag{22}$$

式中 α、θ 分别为相对论电子运动纵向相位和横向相位; $\omega_0=k_0c$, $v_\perp$ 为相对论电子横向速度。C_x、C_y、C_z 为三个积分常数。

在无微扰情况下($\boldsymbol{E}_r=\boldsymbol{B}_r=0$), 相对论电子的横向速度的二个分量可由(20)、(21)式可得

$$v_{xs}=-\frac{2|e|E_{f0}}{m_0\omega_f\gamma_s}\cos\xi_{fs}-\frac{c|e|B_0}{m_0\omega_0\gamma_s}\cos k_0z_s \tag{23}$$

$$v_{ys}=\frac{2|e|E_{f0}}{m_0\omega_f\gamma_s}\sin\xi_{fs}-\frac{c|e|B_0}{m_0\omega_0\gamma_s}\sin k_0z_s \tag{24}$$

式中 $\xi_{fs}=k_fz_s-\omega_ft+\Psi_f$, $z_s=z_0+ut$, u 是相对论电子纵向初速度; $\gamma=\gamma_s$(无微扰条件下)。当 $E_{r0}\neq0$ 时辐射场对相对论电子产生微扰作用,此时 γ 可由下式表示

$$\gamma(t)=\gamma_s+\delta\gamma_1+\delta\gamma_2+\cdots\cdots \tag{25}$$

式中 $\delta\gamma_1$ 和 $\delta\gamma_2$ 分别为相对论因子的一级小量和二级小量(本文忽略更高级小量计算), 由微扰作用下相对论因子的位置坐标为 $x=x_s+\delta x$, $y=y_s+\delta y$, $z=z_s+\delta z$, δx、δy、δz 称为坐标的三个漂移小量,则

$$\begin{aligned}E_{fx}&=E_{f0}\sin[k_f(z_s+\delta z)-\omega_ft+\Psi_f]\\&\approx E_{f0}(\sin\xi_{fs}+k_f\delta z\cos\xi_{fs})\end{aligned} \tag{26}$$

$$\begin{aligned}E_{fy}&=E_{f0}\cos[k_f(z_s+\delta z)-\omega_ft+\Psi_f]\\&\approx E_{f0}(\cos\xi_{fs}-k_f\delta z\sin\xi_{fs})\end{aligned} \tag{27}$$

$$\begin{aligned}E_{rx}&=E_{r0}\cos[k_r(z_s+\delta z)-\omega_rt+\Psi_r]\\&\approx E_{r0}(\cos\xi_{rs}-k_r\delta z\sin\xi_{rs})\end{aligned} \tag{28}$$

$$\begin{aligned}E_{ry}&=-E_{r0}\sin[k_r(z_s+\delta z)-\omega_rt+\Psi_r]\\&\approx -E_{r0}(\sin\xi_{rs}+k_r\delta z\cos\xi_{rs})\end{aligned} \tag{29}$$

相应的相对论电子横向速度二个分量

$$v_x=-\frac{2\Omega_fc}{\omega_f}(\cos\xi_{fs}-k_f\delta z\sin\xi_{fs})-\frac{\Omega_0c}{\omega_f}(\cos k_0z_r-k_0\delta z\sin k_0z_s) \tag{30}$$

$$v_y=\frac{2\Omega_fc}{\omega_f}(\sin\xi_{fs}+k_f\delta z\cos\xi_{fs})-\frac{\Omega_0c}{\omega_0}(\sin k_0z_s+k_0\delta z\cos k_0z_s) \tag{31}$$

式中 $\Omega_f=\frac{|e|E_{f0}}{\gamma_sm_0c}$, $\Omega_0=\frac{|e|B_0}{\gamma_sm_0}$, $\xi_{rs}=k_rz_s-\omega_rt+\Psi_r$。由(15)式得

$$\frac{d}{dt}\gamma(t)=-\frac{|e|}{mc^2}[v_xE_x+v_yE_y]$$

$$
\begin{aligned}
&=\frac{\Omega_0\Omega_f\gamma_s}{\omega_0}\sin(\Delta\omega_{0f}t+\theta_f)\\
&+\frac{\Omega_0\Omega_r\gamma_s}{\omega_0}\cos(\Delta\omega_{0r}t+\theta_r)\\
&+\frac{2\Omega_f\Omega_r\gamma_s}{\omega_f}\cos(\Delta\omega_{fr}t+\theta_{fr})\\
&+\frac{\Omega_0\Omega_f\gamma_s(k_0+k_f)}{\omega_0}\cos(\Delta\omega_{0f}t+\theta_f)\delta z\\
&-\frac{\Omega_0\Omega_r\gamma_s(k_0+k_r)}{\omega_0}\sin(\Delta\omega_{0r}t+\theta_r)\delta z\\
&-\frac{2\Omega_f\Omega_r\gamma_s(k_f-k_r)}{\omega_f}\sin(\Delta\omega_{fr}t+\theta_{fr})\delta z\\
&-\frac{\Omega_0\Omega_f\gamma_s k_0k_f}{\omega_0}\sin(\Delta\omega_{0f}t+\theta_f)\delta z^2\\
&-\frac{\Omega_0\Omega_r\gamma_s k_0k_r}{\omega_0}\cos(\Delta\omega_{0r}t+\theta_r)\delta z^2\\
&+\frac{2\Omega_f\Omega_r\gamma_s k_rk_f}{\omega_f}\cos(\Delta\omega_{fr}t+\theta_{fr})\delta z^2
\end{aligned}
\tag{32}
$$

式中

$$
\left.
\begin{aligned}
\Delta\omega_{0f}&=(\omega_f+\omega_0)\beta_z^0-\omega_f\\
\theta_f&=(k_f+k_0)z_0+\Psi_f\\
\Delta\omega_{0r}&=(\omega_r+\omega_0)\beta_z^0-\omega_r\\
\theta_r&=(k_r+k_0)z_0+\Psi_r\\
\Delta\omega_{fr}&=(\omega_f-\omega_r)\beta_z^0-(\omega_f-\omega_r),\\
\theta_{fr}&=(k_f-k_r)z_0+\Psi_f-\Psi_r\\
\Delta\omega_{0f}&=\Delta\omega_{0r}=\Delta\omega_{fr},\ \ \theta_f-\theta_r=\theta_{fr}\\
\beta_z^0&=\frac{u}{c},\qquad \Omega_r=\frac{|e|E_{r0}}{\gamma_s m_0c}
\end{aligned}
\right\}
\tag{33}
$$

根据 (25) 式，我们有 $\frac{d}{dt}\gamma(t)\approx\frac{d}{dt}\delta\gamma_1+\frac{d}{dt}\delta\gamma_2$，由于(32)式中 δz^2 为更高级小量，可忽略不计。则得

$$
\begin{aligned}
\frac{d}{dt}\delta\gamma_1=&\frac{\Omega_0\Omega_f\gamma_s}{\omega_0}\sin(\Delta\omega_{0f}t+\theta_f)\\
&+\frac{\Omega_0\Omega_r\gamma_s}{\omega_0}\cos(\Delta\omega_{0r}t+\theta_s)\\
&+\frac{2\Omega_f\Omega_r\gamma_s}{\omega_f}\cos(\Delta\omega_{fr}t+\theta_{fr})
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
\frac{d}{dt}\delta\gamma_2=&\frac{\Omega_0\Omega_f\gamma_s(k_0+k_f)}{\omega_0}\cos(\Delta\omega_{0f}t+\theta_f)\delta z\\
&-\frac{\Omega_0\Omega_r\gamma_s(k_0+k_r)}{\omega_0}\sin(\Delta\omega_{0r}t+\theta_r)\delta z\\
&-\frac{2\Omega_f\Omega_r\gamma_s(k_f-k_r)}{\omega_f}\sin(\Delta\omega_{fr}t+\theta_{fr})\delta z
\end{aligned}
\tag{35}
$$

(35)式中 δz 可利用下列关系求得

$$
\begin{aligned}
\frac{d}{dt}[\gamma(1-\beta_z)]=&\frac{d}{dt}\Big[(\gamma_s+\delta\gamma_1)\Big(1-\beta_z^0-\frac{1}{c}\frac{d}{dt}\delta z\Big)\Big]\\
=&(1-\beta_z^0)\frac{d}{dt}\delta\gamma_1-\frac{1}{c}\frac{d}{dt}\delta z\frac{d}{dt}\delta\gamma_1\\
&-\frac{1}{c}(\gamma_s+\delta\gamma_1)\frac{d^2}{dt^2}\delta z
\end{aligned}
\tag{36}
$$

另一方面有

$$
\begin{aligned}
\frac{d}{dt}[\gamma(1-\beta_z)]=&-\frac{|e|}{m_0c}\boldsymbol{\beta}\cdot\boldsymbol{E}-\frac{d}{dt}(\gamma\beta_z)\\
=&\frac{|e|B_0}{m_0c}(c\beta_x\sin k_0z-c\beta_y\cos k_0z)\\
\approx&-\frac{2\gamma_s\Omega_0\Omega_r}{\omega_r}\cos(\Delta\omega_{0r}t+\theta_r)\\
&-\frac{2\gamma_s\Omega_0\Omega_f}{\omega_f}\sin(\Delta\omega_{0f}t+\theta_f
\end{aligned}
\tag{37}
$$

(36) 式中 $\frac{d}{dt}\delta z\frac{d}{dt}\delta\gamma_1$ 和 $\delta\gamma_1\frac{d^2}{dt^2}\delta z$ 二项为更高级小量，可忽略不计，利用(36)、(37)式可得

$$
\begin{aligned}
\frac{d^2}{dt^2}\delta z=&\frac{c(1-\beta_z^0)}{\gamma_s}\cdot\frac{d}{dt}\delta\gamma_1\\
&+\frac{2\Omega_0\Omega_rc}{\omega_r}\cos(\Delta\omega_{0r}t+\theta_r)\\
&+\frac{2\Omega_0\Omega_fc}{\omega_f}\sin(\Delta\omega_{0f}t+\theta_f)
\end{aligned}
\tag{38}
$$

把(34)式代入(38)式并对 t 积分二次得

$$
\begin{aligned}
\delta z=&\frac{2c(1-\beta_z^0)\Omega_f\Omega_r}{\omega_r\Delta\omega_{fr}^2}[\cos\theta_{fr}\\
&-\cos(\Delta\omega_{fr}t+\theta_{fr})-\Delta\omega_{fr}t\sin\theta_{fr}]\\
&+\frac{c\Omega_0\Omega_r}{\Delta\omega_{0r}^2}\Big(\frac{2}{\omega_r}+\frac{1-\beta_r^0}{\omega_0}\Big)(\cos\theta_r\\
&-\cos(\Delta\omega_{0r}t+\theta_r)-\Delta\omega_{0r}t\sin\theta_r)\\
&-\frac{c\Omega_0\Omega_f}{\Delta\omega_{0f}^2}\Big(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\Big)\\
&\times[\sin(\Delta\omega_{0f}t+\theta_f)-\sin\theta_f\\
&-\Delta\omega_{0f}t\cos\theta_f]
\end{aligned}
\tag{39}
$$

(39)代入(35)得

$$
\frac{d}{dt}\delta\gamma_2=\frac{2\Sigma_{0r}H_{fr}}{\omega_0\omega_f\Delta\omega_{fr}}\cos(\Delta\omega_{0f}t+\theta_f)
$$

$$\times(\cos\theta_{fr}-\cos(\Delta\omega_{fr}t+\theta_{fr})-\Delta\omega_{fr}t\sin\theta_{fr})$$
$$+\frac{\Sigma_{0f}H_{0r}}{\omega_0\Delta\omega_{0r}(1-\beta_z^0)}\left(\frac{2}{\omega_r}+\frac{1-\beta_z^0}{\omega_0}\right)$$
$$\times\cos(\Delta\omega_{0f}t+\theta_f)(\cos\theta_r-\cos(\Delta\omega_{0r}t+\theta_r)$$
$$-\Delta\omega_{0r}t\sin\theta_r)$$
$$-\frac{\Sigma_{0f}H_{0f}}{\omega_0\Delta\omega_{0f}(1-\beta_z^0)}\left(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\right)$$
$$\times\cos(\Delta\omega_{0f}t+\theta_f)(\sin(\Delta\omega_{0f}t+\theta_f)$$
$$-\sin\theta_f-\Delta\omega_{0f}t\cos\theta_f)$$
$$-\frac{2\Sigma_{0r}H_{fr}}{\omega_0\omega_f\Delta\omega_{fr}}\sin(\Delta\omega_{0r}t+\theta_r)(\cos\theta_{fr}$$
$$-\cos(\Delta\omega_{fr}t+\theta_{fr})-\Delta\omega_{fr}t\sin\theta_{fr})$$
$$-\frac{\Sigma_{0r}H_{0r}}{\omega_0\Delta\omega_{0r}(1-\beta_r^0)}\left(\frac{2}{\omega_r}+\frac{1-\beta_r^0}{\omega_0}\right)$$
$$\times\sin(\Delta\omega_{0r}t+\theta_r)(\cos\theta_r-\cos(\Delta\omega_{0r}t+\theta_r)$$
$$-\Delta\omega_{0r}t\sin\theta_r)+\frac{\Sigma_{0r}H_{0f}}{\omega_0\Delta\omega_{0f}(1-\beta_z^0)}$$
$$\times\left(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\right)\sin(\Delta\omega_{0r}t+\theta_r)$$
$$\times(\sin(\Delta\omega_{0f}t+\theta_f)-\sin\theta_f-\Delta\omega_{0f}t\cos\theta_f)$$
$$-\frac{4\Sigma_{fr}H_{fr}}{\omega_f^2\Delta\omega_{fr}}\sin(\Delta\omega_{fr}t+\theta_{fr})(\cos\theta_{fr}$$
$$-\cos(\Delta\omega_{fr}t+\theta_{fr})-\Delta\omega_{fr}t\sin\theta_{fr})$$
$$-\frac{2\Sigma_{fr}H_{0r}}{\omega_f\Delta\omega_{0r}(1-\beta_z^0)}\left(\frac{2}{\omega_r}+\frac{1-\beta_z^0}{\omega_0}\right)$$
$$\times\sin(\Delta\omega_{fr}t+\theta_{fr})(\cos\theta_r-\cos(\Delta\omega_{0r}t+\theta_r)$$
$$-\Delta\omega_{0r}t\sin\theta_r)+\frac{2\Sigma_{fr}H_{0f}}{\omega_f\Delta\omega_{0f}(1-\beta_z^0)}$$
$$\times\left(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\right)\sin(\Delta\omega_{fr}t+\theta_{fr})$$
$$\times(\sin(\Delta\omega_{0f}t+\theta_f)-\sin\theta_f-\Delta\omega_{0f}t\cos\theta_f) \tag{40}$$

式中

$$\left.\begin{aligned}\Sigma_{0f}&=\gamma_s\Omega_0\Omega_f(k_0+k_f)\\ H_{0f}&=\frac{\Omega_0\Omega_f c(1-\beta_z^0)}{\Delta\omega_{0f}}\\ \Sigma_{0r}&=\gamma_s\Omega_0\Omega_r(k_0+k_r)\\ H_{0r}&=\frac{\Omega_0\Omega_r c(1-\beta_z^0)}{\Delta\omega_{0r}}\\ \Sigma_{fr}&=\gamma_s\Omega_f\Omega_r(k_f-k_r)\\ H_{fr}&=\frac{\Omega_f\Omega_r c(1-\beta_z^0)}{\Delta\omega_{fr}}\end{aligned}\right\} \tag{41}$$

四、增益讨论

自由电子激光器增益G可表示为

$$G=-\langle\Delta\gamma\rangle_{\psi_r\psi_f}m_0c^2 4\pi\rho_e V/E^2 \tag{42}$$

式中ρ_e是相对论电子束的密度；V为相互作用体积；$\langle\Delta\gamma\rangle_{\psi_r\psi_f}$为电子能量变量对初相位$\Psi_r$和$\Psi_f$的平均值，即

$$\begin{aligned}\langle\Delta\gamma\rangle_{\psi_r\psi_f}&=\langle\gamma(t)-\gamma_s\rangle_{\psi_r\psi_f}\\ &=\langle\delta\gamma_1\rangle_{\psi_r\psi_f}+\langle\delta\gamma_2\rangle_{\psi_r\psi_f}\end{aligned} \tag{43}$$

$\langle\delta\gamma_1\rangle_{\psi_r\psi_f}$和$\langle\delta\gamma_2\rangle_{\psi_r\psi_f}$分别为相对论因子的一级小量和二级小量对初相位$\Psi_r$和$\Psi_f$的平均值。由(34)式得

$$\begin{aligned}\delta\gamma_1=&\frac{\gamma_s\Omega_0\Omega_f}{\omega_0\Delta\omega_{0f}}[\cos\theta_f-\cos(\Delta\omega_{0f}t+\theta_f)]\\ &+\frac{\gamma_s\Omega_0\Omega_r}{\omega_0\Delta\omega_{0r}}[\sin(\Delta\omega_{0r}t+\theta_r)-\sin\theta_r]\\ &+\frac{2\gamma_s\Omega_f\Omega_r}{\omega_f\Delta\omega_{fr}}[\sin(\Delta\omega_{fr}t+\theta_{fr})-\sin\theta_{fr}]\end{aligned} \tag{44}$$

则有

$$\begin{aligned}\langle\delta\gamma_1\rangle_{\psi_r}&=\frac{1}{2\pi}\int_{-\pi}^{\pi}\delta\gamma_1 d\Psi_r\\ &=\frac{\gamma_s\Omega_0\Omega_f}{\omega_0\Delta\omega_{0f}}[\cos\theta_f-\cos(\Delta\omega_{0f}t+\theta_f)]\end{aligned} \tag{45}$$

$$\langle\delta\gamma_1\rangle_{\psi_r\psi_f}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\langle\delta\gamma_1\rangle_{\psi_r}d\Psi_f=0 \tag{46}$$

(45)式表明，在一阶近似条件下，相对论电子与辐射场的相互作用结果与电子运动的自有场相位有关，当$\theta_f=0$时相互作用结果是电子损失能量，即相对论电子处于辐射能量状态；当$\theta_f=\pm\pi$时结果是相对论电子吸收能量，即相对论电子处于被加速状态；当$\theta_f=\pm\frac{\pi}{2}$时结果是相对论电子有时释放能量有时吸收能量(见图1)。由于相对论电子的自有场初相位Ψ_f可在$-\pi\sim\pi$内取任意值，因此必须在$-\pi\sim\pi$内取平均值。(46)式表示在一阶近似条件下，相对论电子与辐射场相互作用，它的能量平均起来没有变化，即电子既没

有获得能量，也没有失去能量。从图 1 也能说明这一点。该结论与忽略自有场情况下所得结果相一致。

自由电子激光器增益可由二级能量变化量获得。我们对(40)式作同样相应的处理，可得

$$\langle\delta\gamma_2\rangle_{\psi_r}=-\frac{\Sigma_{0f}H_{0f}}{\omega_0\Delta\omega_{0f}^2(1-\beta_z^0)}\left(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\right)\Big[1-\cos\Delta\omega_{0f}t+\frac{1}{4}(\cos 2\theta_f-\cos 2(\Delta\omega_{0f}t+\theta_f))-\frac{1}{2}\Delta\omega_{0f}t(\sin(\Delta\omega_{0f}t+2\theta_f)+\sin\Delta\omega_{0f}t)\Big]-\frac{\Sigma_{0r}H_{fr}}{\omega_0\omega_f\Delta\omega_{fr}\Delta\omega_{0r}}\times\Big[\left(1-\frac{\Delta\omega_{fr}}{\Delta\omega_{0r}}\right)(\cos\theta_f-\cos(\Delta\omega_{0r}t+\theta_f))-\frac{\Delta\omega_{0r}}{\Delta\omega_{0f}}(\cos\theta_f-\cos(\Delta\omega_{0f}t+\theta_f))+\Delta\omega_{fr}t\sin(\Delta\omega_{0r}t+\theta_f)\Big]-\frac{\Sigma_{0r}H_{0r}}{\omega_0\Delta\omega_{0r}^2(1-\beta_z^0)}\left(\frac{2}{\omega_r}+\frac{1-\beta_z^0}{\omega_0}\right)\left(1-\cos\Delta\omega_{0r}t-\frac{1}{2}\Delta\omega_{0r}t\sin\Delta\omega_{0r}t\right)-\frac{4\Sigma_{fr}H_{fr}}{\omega_f^2\Delta\omega_{fr}^2}\left(1-\cos\Delta\omega_{fr}t-\frac{1}{2}\Delta\omega_{fr}t\sin\Delta\omega_{fr}t\right)-\frac{\Sigma_{fr}H_{0r}}{\omega_f\Delta\omega_{0r}\Delta\omega_{fr}(1-\beta_z^0)}\left(\frac{2}{\omega_r}+\frac{1-\beta_z^0}{\omega_0}\right)\times\Big[\left(1-\frac{\Delta\omega_{0r}}{\Delta\omega_{fr}}\right)(\cos\theta_f-\cos(\Delta\omega_{fr}t+\theta_f))-\frac{\Delta\omega_{fr}}{\Delta\omega_{0f}}(\cos\theta_f-\cos(\Delta\omega_{0f}t+\theta_f))+\Delta\omega_{0r}t\sin(\Delta\omega_{fr}t+\theta_f)\Big]\tag{47}$$

$$\langle\delta\gamma_2\rangle_{\psi_r\psi_f}=-\frac{\Sigma_{0f}H_{0f}}{\omega_0\Delta\omega_{0f}^2(1-\beta_z^0)}\left(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\right)\left(1-\cos\Delta\omega_{0f}t-\frac{1}{2}\Delta\omega_{0f}t\sin\Delta\omega_{0f}t\right)-\frac{\Sigma_{0r}H_{0r}}{\omega_0\Delta\omega_{0r}^2(1-\beta_z^0)}\left(\frac{2}{\omega_r}+\frac{1-\beta_z^0}{\omega_0}\right)\left(1-\cos\Delta\omega_{0r}t-\frac{1}{2}\Delta\omega_{0r}t\sin\Delta\omega_{0r}t\right)-\frac{4\Sigma_{fr}H_{fr}}{\omega_f^2\Delta\omega_{fr}^2}\left(1-\cos\Delta\omega_{fr}t-\frac{1}{2}\Delta\omega_{fr}t\sin\Delta\omega_{fr}t\right)\tag{48}$$

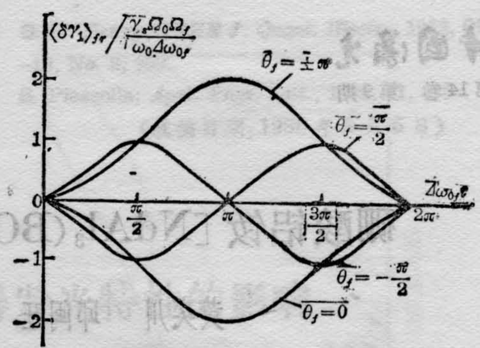

图 1 在一阶近似条件下，相对论电子能量一阶小量与 θ_f 相位之间的关系曲线

由(48)、(46)、(43)和(42)式得增益

$$G(t)=G_0\Big[\frac{\gamma_s\Omega_0^2\Omega_f^2(k_0+k_f)c}{\omega_0\Delta\omega_{0f}^3}\left(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\right)\left(1-\cos\Delta\omega_{0f}t-\frac{1}{2}\Delta\omega_{0f}t\sin\Delta\omega_{0f}t\right)+\frac{\gamma_s\Omega_0^2\Omega_r^2(k_0+k_r)c}{\omega_0\Delta\omega_{0r}^3}\left(\frac{2}{\omega_r}+\frac{1-\beta_z^0}{\omega_0}\right)\left(1-\cos\Delta\omega_{0r}t-\frac{1}{2}\Delta\omega_{0r}t\sin\Delta\omega_{0r}t\right)+\frac{4\gamma_s\Omega_f^2\Omega_r^2(k_f-k_r)c}{\omega_f^2\Delta\omega_{fr}^3}\left(1-\cos\Delta\omega_{fr}t-\frac{1}{2}\Delta\omega_{fr}t\sin\Delta\omega_{fr}t\right)\Big]\tag{49}$$

式中 $G_0=m_0c^2 4\pi\rho_e V/E^2$。如果我们忽略电子运动本身的自有场影响，即假定 $E_{f0}=0$，同时考虑实际 $\omega_r\gg\omega_0$ 情况，我们也可以从(49)式直接得出通常文献中所给出的自由电子激光器增益[8]

$$G=\frac{4e^4B_0^2\lambda_0\rho_e V}{c(m_0\gamma_s\Delta\omega_{0r})^3}\left(1-\cos\Delta\omega_{0r}t-\frac{1}{2}\Delta\omega_{0r}t\sin\Delta\omega_{0r}t\right)\tag{50}$$

从(49)式我们可看出，带电粒子的自有场对自由电子激光器的增益是有影响的。它

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6. 激光输出为线性偏振光

我们用格林棱镜作检偏器，测出激光输出的偏振度 $P_d=1$。故 NAB 晶体的激光输出为线性偏振光。

7. 激光输出稳定性

NAB 晶体的激光输出随时间变化规律如何(即所谓的使用寿命)是一个重要参数。我们用 $\phi 1.9\times 9.5\,\mathrm{mm}^3$ 的 NAB 晶体进行了寿命实验。氙灯尺寸为 $\phi 2.5\times 20\,\mathrm{mm}^3$，谐振腔长为 140 mm，全反镜 $R=100\%$，输出镜 $T=50\%$，输入电容 $13\,\mu\mathrm{F}$，输入电压 850V，测得激光脉冲输出能量 $W_{out}=13.4\,\mathrm{mJ}$。重复频率 45 次/小时，连续实验 22 天，共发射激光脉冲 23760 次，未见激光衰减。这说明，NAB 的脉冲激光输出是相当稳定的。

四、结　　论

实验结果表明，我所研制的 NAB 晶体具有激光泵浦阈值低、增益大、效率高、输出线性偏振光、光束发散角小以及激光性能稳定等优点，是一种优质的小型固体激光材料。以它做成的小型激光器件，重量轻、体积小、成本低，对国防、军工、工业、民用、教学和科研等领域都有实用价值。如能找到波长匹配的高强度发光二极管或激光二极管泵浦，则这种激光器在激光通讯、光纤传感和集成光学等方面也可能具有一定的适用性。

在 NAB 晶体的测试过程中，得到西安五机部 205 所廉汝林、张小舟、郭慧敏等同志，上海交通大学应用物理系方书淦、李胜华等老师，江苏地质矿产局中心实验室赵炎生、阮雍生等同志的大力支持和帮助，谨在此一并致以诚挚的谢意。

参　考　文　献

[1] H. D. Hattendorff *et al.*; *J. Phys. C: Solid State Phys.*, 1977, **11**, 2399.

[2] S. R. Chinn, H. Y-P Hong; *Opt. Commun.*, 1975, **15**, No. 3, 345.

[3] G. Huber, H. G. Danielmeyer; *Appl. Phys.*, 1979, **18**, No. 1, 77.

[4] Gerhard Winzer *et al.*; *IEEE J. Quant. Electr.*, 1978, **QE-14**, No. 11, 840.

[5] V. L.Bilak *et al.*; *Sov. Phys. Dokl.*, 1978, **23**, No. 5, 199.

[6] S. R. Chinn *et al.*; *Laser Focus*, 1976, **12**, No. 5, 64.

[7] Walter Koechner; "Solid State Laser Engineering", Springer-Varlag, 1976, p. 74～122.

[8] 纪钟;《激光》, 1981, 8, No, 2, 26.

[9] 王保林等;《硅酸盐学报》, 1984, **12**, No. 3, 259.

[10] Luo Zundu *et al.*; *Chinese Physics Letters*, 1986, **3**, No. 11～12.

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与忽略带电粒子自有场情况相比较，增益表达式增加了二项(第一和第三项)，虽然(49)式的第三项的系数为负数(因 $\dfrac{(k_f-k_r)c}{\Delta\omega_{fr}^3}=\dfrac{1}{c^2(k_f-k_r)^2(\beta_z^0-1)^3}<0$)，但是(49)式的第一项系数和第三项系数相比较，并考虑实际情况中有 $cB_0\gg E_{r0}$。则有

$$G_0\left[\frac{\gamma_s\Omega_0^2\Omega_r^2(k_0+k_f)c}{\omega_0\Delta\omega_{0r}^3}\left(\frac{2}{\omega_f}+\frac{1-\beta_z^0}{\omega_0}\right)-\left|\frac{4\gamma_s\Omega_f^2\Omega_r^2(k_f-k_r)c}{\omega_f^2\Delta\omega_{fr}^3}\right|\right]$$

$$=\frac{e^4E_{0f}^2G_0}{\omega_f\cdot\gamma_s^3m_0^4c}\left[\frac{B_0^2(k_0+k_f)}{\omega_0\Delta\omega_{0f}^3}-\left|\frac{E_{0r}^2(k_f-k_r)}{\omega_fc^2\Delta\omega_{fr}^3}\right|\right]>0。\qquad(51)$$

(51)式说明，所得增益比忽略自有场情况下激光器增益将有一定量的增加。

参　考　文　献

[1] W. B. Colson; *Phys. Lett.*, 1977, **A64**, 190.

[2] A. Yariv, C. Shih; *Opt. Commun.*, 1978, **24**, 233.

[3] A. Bambini, A. Renieri; *Lett Al NUOVO Cimento*, 1978, **21**, 399.

[4] A. Bambini *et al.*; *Phys. Rev.*, 1979, **A19**, 2013.

[5] W. H. Louisell *et al.*; *Phys. Rev.*, 1979, **A19**, 288.

[6] C. Shih, A. Yariv; *Phys. Rev.*, 1980, **A22**, 2717.

[7] 曹昌祺;"电动力学", 人民教育出版社, 1962 年。

[8] W.B.Colson; *Phys. of Quant. Electr*, 1978, **5**, 157.