Walsh transform by the coherent optical method

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The space-variant optical information processing can be performed through spatial filtering of transforms other than Fourier transform. One of the authors and coworkers have demonstrated that the general optical linear transforms can be realized with holographic lenses systems, and such system can be obtained by the iterative method.

Walsh transform is an important linear transform, some authors applied it to optical problems such as image improvement, but they realized Walsh transform with electronic computers. The realization of Walsh transform with optical methods is one of the subjects which have not yet been solved well enough. We consider the complex Walsh functions $Wa|_0$ (j, k) defined as follows

$$Wa|_0\,(o, k)=Wa|_W\,(o, k)$$

$$Wa|_0\,(l, k)=\frac{1}{2}\,[(1-i)\ Sal\ (l, k)+(1+i)\ Ca|\ (l, k)]$$

$$Wa|_0\left(\frac{N}{2}, k\right)=Sa|\left(\frac{N}{2}, k\right)$$

$$Wa|_0\left(\frac{N}{2}+l,\ k\right)Wa|_0^*\left(\frac{N}{2}-l, k\right)$$

$$\begin{cases} l=1, 2, \ldots \left(\frac{N}{2}-1\right) \\ K=\ 0., 1, \ldots\ N-1 \end{cases}$$

where $Wa|_W$ (o, k), Sa| (l, k) and Ca| (l, k) are the Walsh functions in Walsh ordering. We can write $wa|_0$ (j, k) as

$$Wal_0\,(N)=K_nG_0K_{n-1}\ldots\ K_2\ G_0\ K_1 \qquad (1)$$

where N is the order of $Wa|_0$, G_0 is the free space transmission matrix, and K_j (j=1, 2, ... n) is the trasmittance function of the jth holographic lens. Solving eq.(1) with the electronic computer using our iterative method program, we obtained the design of holographic lenses systems, which are necessary for realizing the 8-order complex Walsh transform.

The Kinoforms are served as holographic lenses in our experiment because of their high diffracting efficiency. The phase variations of the Kinoforms were controlled by bleaching method. To ensure the unitarity of the transform, and if we put the sampling separations of all planes are equal and are represented by Δx, the condition $\Delta x=\sqrt{L\lambda/N}$ must be satisfied, where L is the distance between two holographic lenses. Since the transform system requires very high precision of arrangement, we utilized the auto-collimated method by reflection of laser beam, and made use of Talbot effect for adjusting the system. The 8-order $Wa|_w$(o,k) and Sal (l,k) l=1,2,3,4, were regarded as input functions in our transform system, and we obtained the photographs of Walsh spectra at the output plane, which are in agreement with theoretical expectation.

用相干光方法实现 Walsh 变换

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运用非傅里叶变换实现空间滤波，可解决空间变光学信息处理问题。本文作者之一及其合作者曾论证，用全息透镜列可实现一般线性变换，并给出求解方法—迭代法。

Walsh 变换是一种重要的线性变换，有些作者曾把它用于象质改善等光学问题，但他们是籍助于电子计算机来实现 Walsh 变换的。用光学方法实现 Walsh 变换还是没有很好解决的问题。我们考虑如下定义的复数 Walsh 函数 $Wa|_0(j, k)$：

$$Wa|_0(0, k) = Wa|_W(0, k)$$

$$Wa|_0(l, k) = \frac{1}{2}[(1-i)\,Sa|(l, k) + (1+i)Ca|(l, k)]$$

$$Wa|_0\left(\frac{N}{2}, k\right) = Sa|\left(\frac{N}{2}, k\right)$$

$$Wa|_0\left(\frac{N}{2}+l, k\right) = Wa|^*_0\left(\frac{N}{2}-l, k\right)$$

$$\begin{cases} l = 1, 2, \cdots \left(\frac{N}{2}-1\right) \\ k = 0, 1, \cdots N-1 \end{cases}$$

式中 $Wa|_W(0, k)$、$Sa|(l, k)$和 $Ca|(l, k)$是 Walsh 序的 Walsh 函数。我们可将复数Walsh 函数表为：

$$Wa|_0(N) = K_n G_0 K_{n-1} \cdots K_2 G_0 K_1 \tag{1}$$

式中N表示 $Wa|_0$ 的序数，G_0 表示自由空间传播矩阵，而 $K_j(j=1, 2, \cdots n)$表示第 j 个全息透镜的透射率函数。我们编写了迭代法求解的计算机程序，求出实现 8 序复数 Walsh 变换所需的全息透镜列。

我们采用 Kinoform 作为全息透镜，因为它具有高衍射能力，用乳胶漂白方法控制 Kinoform 上位相变化。为了保证变换的公正性，而且让所有平面的取样间隔相等并表示为 Δx，则应满足 $\Delta x = \sqrt{L\lambda/N}$，其中 L 是两相邻平间的距离。由于全息透镜列排列精度要求很高，我们应用激光束反射自准方法和利用 Talbot 效应调准变换系统。用 8 序 $Wa|_W(0, k)$和 $Sa|(l, k)$ $l=1, 2, 3, 4$ 作为输入函数，在输出平面摄得 Walsh 谱的照片与理论预期一致。